

TOPICAL REVIEW

# Entanglement entropy from a holographic viewpoint

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**Abstract.** The entanglement entropy has been historically studied by many authors in order to obtain quantum mechanical interpretations of the gravitational entropy. The discovery of AdS/CFT correspondence leads to the idea of holographic entanglement entropy, which is a clear solution to this important problem in gravity. In this article, we would like to give a quick survey of recent progresses on the holographic entanglement entropy. We focus on its gravitational aspects, so that it is comprehensible to those who are familiar with general relativity and basics of quantum field theory.

## 1. Introduction

One of the most mysterious and fascinating aspects in general relativity is the existence of black holes. They are peculiar only to theories with gravity. Among all important properties, a black hole has its own entropy, given by the Bekenstein-Hawking formula

$$S_{BH} = \frac{\text{Area}(\Sigma)}{4G_N}, \quad (1)$$

where  $\Sigma$  is the horizon and  $G_N$  is the Newton constant [1]. This formula (1) suggests us that the degrees of freedom contained in a certain region in gravity is actually proportional to its area instead of the volume. This observation developed into the idea of holography (or holographic duality) [2]. The holographic principle argues that a gravitational theory in a  $d + 2$  dimensional spacetime  $M$  is equivalent to a non-gravitational theory on a  $d + 1$  dimensional spacetime  $\partial M$ , which is the boundary of  $M$ . The latter theory is typically described by a quantum many-body system. A concrete example of holography was later obtained in string theory and this is called the AdS/CFT correspondence [3, 4] (for a review see [5]). This is the particular case of holography where the gravity lives in a spacetime with a negative cosmological constant.

The original Bekenstein-Hawking formula (1) relates the area of horizon, which is a geometric data, to the entropy, which is a quantum mechanical quantity. This correspondence between a geometric quantity and a microscopic data is the key concept of holography. Therefore it is natural to expect such relations for more general observables. In particular, we can ask what is the holographic dual of the areas of more general surfaces in a gravitational theory. There has been progresses in this direction recently by employing the idea of holography. The upshot is that the area of a minimal surface in a (Euclidean) gravitational theory corresponds to the entanglement entropy in its dual non-gravitational theory [6, 7]. This is simply summarized into the formula

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}, \quad (2)$$

where  $S_A$  is the entanglement entropy for the subsystem  $A$ , and  $\gamma_A$  is a codimension two minimal area surface whose boundary  $\partial\gamma_A$  coincides with  $\partial A$ . This is called the holographic entanglement entropy. Intuitively, the entanglement entropy  $S_A$  measures how much information is hidden inside  $B$ , when we divide the total space into two parts  $A$  and  $B$  (we will give a precise definition later).

Notice that the minimal area surfaces are more general than horizons of static black holes because in the former the trace of extrinsic curvature is required to vanish, while in the latter each component of extrinsic curvature should be vanishing. Later, this holographic entanglement entropy is covariantly generalized into the case where the spacetime is Lorentzian, which can be time-dependent in general [8]. See the earlier review articles [9] for a comprehensive review of holographic entanglement entropy. Refer to [10] for a detailed review on connections between the entanglement entropy and the entropy of black holes. Also a brief introduction of the holographic entanglement entropy

is can be found in the review article [11] on the application of holography to condensed matter physics.

Historically, the entanglement entropy has originally been introduced to quantum field theories in the attempt to understand the microscopic origin of black hole entropy [12, 13, 14, 10, 15, 16, 17, 18]. The entanglement entropy has first been studied by using AdS/CFT setups with horizons in the pioneering works [19, 20].

The purpose of this article is to review recent progresses on the holographic entanglement entropy especially focusing on the gravitational dynamics such as black hole formations. For example, a time evolution of a black hole can be quantitatively measured as the evolution of holographic entanglement entropy. Even though the definition of horizon entropy is ambiguous in time-dependent black hole backgrounds depending on the choice of a time slice, the definition of the holographic entanglement entropy is unique [8]. This is one of the remarkable advantages of the holographic entanglement entropy. We will explain more details of these later.

The entanglement entropy offers us an important observable when the spacetime  $M$  has an additional boundary which intersects its boundary  $\partial A$ . In this case, the non-gravitational theory lives on a manifold with a boundary. In the context of AdS/CFT, such a situation occurs when the conformal field theory (CFT) is defined on a manifold with a boundary, called the boundary conformal field theory (BCFT). Recently, the entanglement entropy has been computed in this AdS/BCFT setup and has been shown to characterize the BCFT, as we will review later. In this way, the entanglement entropy is useful when we would like to characterize a gravitational spacetime with a non-trivial topology.

In the present article, we will only give a minimum guide to the entanglement entropy in quantum many-body systems, which nevertheless suffices to understand the rest of the material. Refer to [21, 22, 23] for the review papers on entanglement entropy in quantum field theories and to [24, 25, 26] for those in quantum many-body systems. We would also like to mention that the entanglement entropy has recently been applied to condensed matter physics as a new order parameter which classifies various quantum phases, though we will not discuss this aspect here.

This article is organized as follows: In section 2, we explain the definition and basic properties of entanglement entropy. Later we explain the holographic entanglement entropy based on the AdS/CFT. We will give a brief introduction to the AdS/CFT. In section 3, we discuss the holographic entanglement entropy in the presence of black holes. We also explain the analysis of black hole formations by employing the holographic entanglement entropy. In section 4, we explain the holographic dual of CFT on a manifold with a boundary. In section 5, we summarize conclusions and discuss future directions.

## 2. Holographic Entanglement Entropy from AdS/CFT

Here we first introduce the basic definition and properties of the entanglement entropy in quantum many-body systems. After that we will explain the holographic entanglement entropy with a brief introduction to the AdS/CFT correspondence.

### 2.1. Definition and Properties of Entanglement Entropy

A state in quantum mechanics is described by a vector  $|\Psi\rangle$  in a Hilbert space  $\mathcal{H}$ , which evolves in time by its Hamiltonian  $H$ . Let us assume that the quantum system we consider has multiple degrees of freedom (e.g. the quantum mechanics for more than one particles) so that we can decompose the total system into two subsystems  $A$  and  $B$ . Accordingly, the total Hilbert space  $\mathcal{H}$  becomes a direct products

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B. \quad (3)$$

For example, we can consider a spin chain, where a lot of spins are arrayed in a line as in Fig.1. In this example, we can choose  $A$  and  $B$  in many different ways just by cutting the chain at an arbitrary point.

In quantum mechanics, physical quantities are computed as expectation values of operators as follows

$$\langle O \rangle = \langle \Psi | O | \Psi \rangle = \text{Tr}[\rho \cdot O], \quad (4)$$

where we defined the density matrix  $\rho = |\Psi\rangle\langle\Psi|$ . This system is called a pure state as it is described by a unique wave function  $|\Psi\rangle$ . In more general cases, called mixed states, the system is described by a density matrix  $\rho$  as in (4) instead of the wave function  $|\Psi\rangle$ , normalized such that  $\text{Tr}\rho = 1$ . A typical example of a mixed state is the canonical distribution  $\rho = e^{-\beta H} / \text{Tr}[e^{-\beta H}]$  at finite temperature  $T = \beta^{-1}$ .

We define the reduced density matrix  $\rho_A$  for the subsystem  $A$  by tracing out with respect to  $\mathcal{H}_B$  by

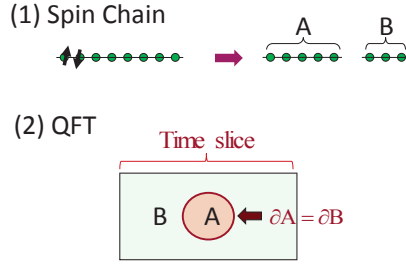
$$\rho_A = \text{Tr}_B[\rho]. \quad (5)$$

Then the entanglement entropy is defined as the von-Neumann entropy for  $\rho_A$

$$S_A = -\text{Tr}[\rho_A \log \rho_A]. \quad (6)$$

In the context of this paper, we consider the entanglement entropy in quantum field theories. We can view a quantum field theory (QFT) as an infinite copies of quantum mechanics. Therefore, its Hilbert space is given by all possible field configurations of QFT at a fixed time. Thus  $\mathcal{H}_A$  is defined as those included in the subspace  $A$  on a fixed time slice<sup>‡</sup>. In this way we can geometrically define the subsystem  $A$  as in Fig.1. The choice of  $A$  is uniquely defined in terms of the boundary  $\partial A$ . There are obviously infinitely different definitions of the entanglement entropy  $S_A$  depending on the choices of  $A$ .

<sup>‡</sup> Recently, the entanglement entropy for the subsystem  $A$  by a region in the momentum space has been analyzed in [27].



**Figure 1.** Examples of choices of subsystem  $A$ .

We would like to summarize the basic properties of the entanglement entropy, which are useful in later arguments (refer to [28] for more details). If the total system is a pure state, the equality  $S_A = S_B$  is always satisfied. This means that the entanglement entropy for a pure state is not extensive as opposed to the thermal entropy. Also, for any systems, when we divide the system into four subsystems  $A$ ,  $B$ ,  $C$  and  $D$  so that there are no overlap between each of them i.e.  $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \otimes \mathcal{H}_D$ , the following inequality is always satisfied [29]:

$$S_{A \cup B} + S_{B \cup C} \geq S_{A \cup B \cup C} + S_B. \quad (7)$$

This inequality is called the strong subadditivity [29, 30]. These are properties which are true for any quantum mechanical systems. It was shown in [31] that we can derive the c-theorem from the strong subadditivity if applied to two dimensional relativistic field theories. Recently, an extension of this analysis has been obtained in three dimensional field theories by applying the strong subadditivity to infinitely many subsystems in [32], which offers us an entropic proof of so called F-theorem [33].

One more useful property is the area law for quantum field theories. Since the quantum field theories have infinitely many degrees of freedom, the entanglement entropy  $S_A$  is divergent. It has been shown that the leading divergence term is proportional to the area of the boundary  $\partial A$  [13, 14]:

$$S_A = \gamma \cdot \frac{\text{Area}}{a^{d-1}} + O(a^{-(d-2)}), \quad (8)$$

where  $\gamma$  is a numerical constant;  $a$  is the ultra-violet(UV) cut off in quantum field theories which is proportional to the lattice constant. The continuum limit corresponds to  $a \rightarrow 0$ . This is called the area law (see also e.g.[34, 30]). This has been proved for free field theories [35, 24, 25, 26]. Even though for interacting field theories, there has been no systematic direct test of area law, the holographic calculation using the AdS/CFT [6, 7] implies that the area law is true for any interacting quantum field theories which have UV fixed points. See also [36] for a consistency condition for the entanglement entropy in QFTs.

We should mention that there is an important exception of area law (8). In two dimensional field theories, the area law is violated in a logarithmic way if they are scale invariant. Since scale invariant theories have a conformal symmetry, they are called conformal field theories (CFTs). A simplest example of a CFT is a free massless scalar field theory. In a two dimensional CFT defined on a infinitely extended line, we have the following general result [37, 38]

$$S_A = \frac{c}{3} \log \frac{l}{a}, \quad (9)$$

where  $c$  is the central charge of the CFT;  $l$  is the length of subsystem  $A$ . In this way the  $S_A$  has the logarithmic divergence. For more results on the entanglement entropy in two dimensional CFTs, refer to e.g.[38, 21, 22, 23]. A partial list of the analysis of entanglement entropy in massive field theories or higher dimensional field theories can be found in [39, 40, 41, 42, 43].

## 2.2. Holography and AdS/CFT

The holographic principle argues that a gravitational theory in a  $d + 2$  dimensional spacetime  $M$  is equivalent to a non-gravitational theory on a  $d + 1$  dimensional spacetime  $\partial M$ , which is the boundary of  $M$  [2]. The latter theory is typically described by a quantum many-body system. A concrete example of holography is known as the AdS/CFT correspondence [3, 4]. The AdS/CFT correspondence argues that a gravity on a  $d + 2$  dimensional Anti-de Sitter space ( $\text{AdS}_{d+2}$ ) is equivalent to a CFT on  $d + 1$  dimensional boundary of  $\text{AdS}_{d+2}$ , which is called  $\text{AdS}_{d+2}/\text{CFT}_{d+1}$ . A typical choice of the coordinate of AdS space is the Poincare coordinate, where the metric of  $\text{AdS}_{d+2}$  is given by

$$ds^2 = R^2 \frac{dz^2 + dx^\mu dx_\mu}{z^2}, \quad (10)$$

where  $\mu = 0, 1, \dots, d$ . The parameter  $R$  is called the radius of AdS. In this case, the boundary of  $\text{AdS}_{d+2}$  is given by the spacetime spanned by  $(x^0, x^1, \dots, x^d)$  at  $z = 0$ . Since the metric at  $z = 0$  gets divergent, we need to introduce the cut off as  $z > a$ , using an infinitesimally small constant  $a$ . This cut off in the AdS space is equivalent to the UV cut off  $a$  in CFT up to an order one constant. The important fact is that this new coordinate  $z$  corresponds to the length scale (or inverse of energy scale) of the dual CFT in the sense of RG-flow.

The basic principle of AdS/CFT is called the bulk to boundary relation [4]. This argues that the partition function of CFT is equal to that of the gravity on the AdS space i.e.  $Z_{\text{CFT}} = Z_{\text{AdS}}$ . In the classical gravity limit, which is assumed in the most parts of this article, the gravity partition function is just given in terms of the one-shell action  $I_{\text{AdS}}$  as  $Z_{\text{AdS}} = e^{-I_{\text{AdS}}}$  in the Euclidean signature.

The AdS/CFT correspondence was originally found by considering near horizon geometries of D-branes in string theory [3]. Even though we need to know the details of this in order to precisely identify CFTs which is dual to the AdS spaces with

various radius  $R$ , we will not get into the details as they are not necessarily crucial to understand the concept of AdS/CFT described below. Instead, we would like to ask the readers to refer to string theory literature e.g. the review [5] on this string theoretic understandings. Here we would like to simply mention that the most useful conclusion which can be obtained from the string theory arguments can be summarized as follows. The dual CFTs are usually given by  $SU(N)$  Yang-Mills gauge theories with a ('t Hooft) coupling constant  $\lambda$ , corresponding to  $N$  D-branes. The classical gravity limit (or called supergravity limit) is given by the limit where both  $N$  and  $\lambda$  are taken to be infinitely large. In this limit, the string theory is reduced to the supergravity, which can be regarded as the general relativity coupled to other fields such as the scalar fields and gauge fields. This is because the large  $N$  limit suppresses the quantum gravity effect and the large coupling limit suppresses the string theoretic corrections.

The AdS/CFT can be applied to more general background. We can modify the infrared (IR) geometry i.e. the large  $z$  region. We always require that in the boundary limit  $z \rightarrow 0$ , the metric approaches that of the pure AdS (10), which is called the asymptotically AdS condition. Though it is believed that we can extend the AdS/CFT to more general backgrounds which are not asymptotically AdS, we will not discuss this here.

To understand the AdS/CFT better, it seems very important to study how the information in the CFT is encoded in that in the gravity theory. Especially, we can consider the information included in a certain region  $A$  in the CFT and ask what is dual to it in the AdS gravity. Since the amount of information in the region  $A$  can be measured by the entanglement entropy  $S_A$ , it will be interesting to consider what is the AdS dual of the entanglement entropy in a CFT. To answer this question is the main subject of this article.

### 2.3. Holographic Entanglement Entropy

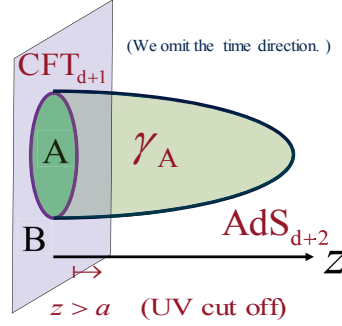
In [6, 7], by applying the AdS/CFT correspondence, it is argued that the entanglement entropy  $S_A$  in a CFT can be holographically calculated by the following formula of holographic entanglement entropy (see Fig.2):

$$S_A = \text{Min}_{\Sigma_A} \left[ \frac{\text{Area}(\Sigma_A)}{4G_N} \right], \quad (11)$$

where  $\Sigma_A$  is a codimension two surface (i.e.  $d$  dimensional in  $\text{AdS}_{d+2}$ ) which satisfies  $\partial\Sigma_A = \partial A$ ; we also require that  $\Sigma_A$  is homologous to  $A$ . The minimum in (11) is taken for all surfaces  $\Sigma_A$  which satisfy this condition. Therefore  $\Sigma_A$  is finally becomes the minimal area surface  $\gamma_A$  as in (2). This formula (11) can be applied to any static setups. The minimal area surface is well-defined in the static case because we can equivalently consider a Euclidean AdS space.

When the background is time-dependent, we need to employ the covariant holographic entanglement entropy [8], which is given by replacing  $\Sigma_A$  with the extremal surface in the Lorentzian asymptotic AdS space which satisfies the previous condition.

This covariant description corresponds to the minimization of the Bousso's covariant entropy bound [45]. If there are several extremal surfaces we take the one with minimum area.



**Figure 2.** The calculation of holographic entanglement entropy.

It is straightforward to see that the holographic entanglement entropy (11) leads to the area law as long as the gravity lives on an asymptotically AdS space, which is dual to a field theory with a UV fixed point. This is because the AdS metric gets divergent at  $z = 0$  and this near boundary region gives the dominant divergent contributions to the area of minimal surface which is obviously proportional to the area of  $\partial A$ .

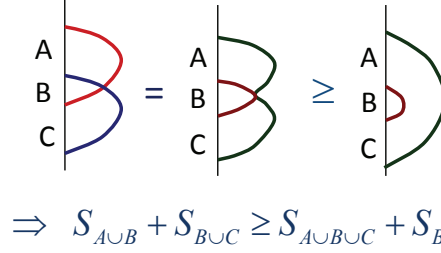
The strong subadditivity (7) can also be holographically proven very quickly for the holographic entanglement entropy in static backgrounds [44] (see also [46]). The essence of this proof is summarized in the Fig.3. This only employs the fact that the holographic entanglement entropy is given by a minimum of a certain integral on the surface  $\Sigma_A$ . Moreover, it was recently found that another inequality called monogamy can be derived in an analogous way [47]. This proves the Cadney-Linden-Winter inequality [48], which is known to be independent from the strong subadditivity and is known to be always satisfied for any quantum systems.

Just to satisfy the strong subadditivity and other inequalities, we can replace the area function with other functionals which include e.g. higher derivatives of curvatures. Indeed, this degrees of freedom needs to be employed to find the holographic entanglement entropy for gravity theories with higher derivative corrections such as the Gauss-Bonnet gravity as we briefly explain later. However, for the Einstein gravity coupled to any matter fields, we should choose the area functional. In the presence of black hole horizon in the AdS space, the minimal surface tends to wrap on the horizon. Thus in order to be consistent with the Bekenstein-Hawking formula (1), we are naturally lead to the area formula (11).

#### 2.4. Evidences

Even though the holographic entanglement entropy formula (11) has not been proven at present, there have been many supporting evidences and have been no counterexamples.





**Figure 3.** The holographic proof of strong subadditivity. In each of three figures, the vertical black line represents the boundary of the AdS, while the horizontal direction in the right is the  $z$  direction in AdS. Though we are assuming the time slice of  $\text{AdS}_3$  just for simplicity, this argument can be extended into higher dimension in a straightforward way. In the left picture, the red and blue curve represents the minimal surfaces  $\gamma_{A \cup B}$  and  $\gamma_{B \cup C}$ . In the middle picture, we just recombine them into two surfaces (green and brown ones). The true minimal surfaces  $\gamma_{A \cup B \cup C}$  and  $\gamma_B$  are given by the right picture. Therefore the strong subadditivity is obvious.

Though a heuristic understanding of the formula (2) was given in [49], this argument is not complete as pointed out in [50] (see also [51]). Thus, instead of giving a proof, below we would like to list some of important evidences.

- As we explained before, we can derive the area law and strong subadditivity from the holographic formula (2).
- We can explicitly confirm that in the  $\text{AdS}_3/\text{CFT}_2$ , the holographic entanglement entropy precisely agrees with the CFT result[6, 7, 52]. This can be seen as follows. We start with the Poincare metric of  $\text{AdS}_3$

$$ds^2 = R^2 \frac{dz^2 - dt^2 + dx^2}{z^2}. \quad (12)$$

The two dimensional CFT lives on the space spanned by  $t$  and  $x$ . We choose the subsystem  $A$  to be the length  $l$  interval  $|x| \leq l/2$  in the infinitely long total space  $-\infty < x < \infty$ . In  $\text{AdS}_3/\text{CFT}_2$ , the minimal surface  $\gamma_A$  is given by a geodesic line in  $\text{AdS}_3$  on a time slice  $t = \text{constant}$ . It is an elementary exercise to see that it is given by a half circle with radius  $l/2$  i.e.  $x = \sqrt{l^2/4 - z^2}$ . Since the induced metric on this geodesic is given by

$$ds_{ind}^2 = \frac{l^2 dz^2}{4z^2 \sqrt{l^2/4 - z^2}}. \quad (13)$$

The holographic entanglement entropy now reads as

$$S_A = \frac{R}{2G_N} \int_a^{l/2} dz \frac{l}{2z \sqrt{l^2/4 - z^2}} = \frac{R}{2G_N} \log \frac{l}{a} = \frac{c}{3} \log \frac{l}{a}. \quad (14)$$

Here we employed the relation  $c = \frac{3R}{2G_N}$  between the central charge of 2 dim. CFT and the radius of  $\text{AdS}_3$  [53]. This precisely agrees with the CFT result (9).

- The proof of holographic formula (2) in the special case where  $A$  is a round sphere has been given in [54] for any dimensions. This analysis has been generalized to calculate the Renyi entropy [55] (see also [56, 57]).
- In the setup of  $\text{AdS}_{d+2}/\text{CFT}_{d+1}$ , for a generic, smooth and compact subsystem  $A$ , the holographic entanglement entropy behaves as follows [6, 7]:

$$S_A = p_1 (l/a)^{d-1} + p_3 (l/a)^{d-3} + \dots \quad (15)$$

$$\dots + \begin{cases} p_{d-1} (l/a) + p_d + \mathcal{O}(a/l) , & d+1: \text{ odd} , \\ p_{d-2} (l/a)^2 + q \log(l/a) + \mathcal{O}(1) , & d+1: \text{ even} . \end{cases}$$

Thus there is a logarithmic divergent term in even dimensional CFTs, This is a universal term in that its coefficient  $q$  is independent from the UV cut off  $a$ . In general,  $q$  is proportional to a linear combination of the central charges in  $\text{CFT}_{d+1}$ . We already explicitly explained that this agrees with the  $\text{CFT}_2$  result. In the higher dimensional cases, the agreement of  $q$  between the  $\text{AdS}_{d+2}$  and  $\text{CFT}_{d+1}$  has been confirmed in [7, 58, 59, 54] (see also [60, 61, 62]).

On the other hand, in odd dimensional CFTs, we find that the finite constant  $p_d$  is independent from the cut off. It has been suggested that  $p_d$  can be used as a measure of degrees of freedom in odd dimensional CFT, where there are no conformal anomalies and central charges [59, 54, 63]. This is expected to be related to the F-theorem in three dimensional CFTs [33]. See also [64, 65] for other relations between the entanglement entropy and RG flow. Refer to [66, 67, 68, 69, 70, 71] for calculations of  $q$  and  $p_d$  in free field theories. See also [72, 73] for the analysis of the entanglement entropy in three dimensional  $O(N)$  vector models and Chern-Simons theories.

The holographic entanglement entropy was recently analyzed in the presence of relevant perturbation in [74], where extra logarithmic contributions have been observed. A similar result has already been obtained in the free scalar field theory [75].

- If we consider the holographic entanglement entropy in gravity duals of confining gauge theories such as the AdS soliton [76] and Klebanov-Strassler solutions [77], we find that the derivative of  $S_A$  with respect to the size of  $A$  gets discontinuous at some point [78, 79, 80, 81, 82, 83, 84]. This is considered to be dual to the confinement/deconfinement phase transition dual to the Hawking-Page transition [85]. The lattice calculations [86, 87] (see also [88]) of pure Yang-Mills theory qualitatively confirm this prediction from AdS/CFT, though the order of phase transition is no longer first order for these finite  $N$  calculations. In particular, it was shown that the holographic entanglement entropy computed for the AdS soliton geometry precisely agrees with that computed in the free field theory [78] when supersym-

metry is only weakly broken. An analogous result is obtained for the geometric entropy in [89, 90]. For studies of holographic entanglement entropy for some other gravity duals including non-conformal theories refer to [91, 92, 93, 94, 95, 96, 97]. It is possible that the entanglement entropy can be a useful probe of QCD [98].

- Consider the case where the subsystem  $A$  consists of disconnected regions e.g.  $A_1 \cup A_2$ . The holographic entanglement entropy predicts phase transitions when we change the distance between  $A_1$  and  $A_2$  [50, 99]. In the 2 dim. CFT, these results have been shown to be consistent with those in CFT [50] in a non-trivial way. For relevant calculations in the CFT side refer to [100, 101, 102, 103].

### 2.5. Higher Derivative Corrections

The holographic formula (2) assumes the classical gravity limit of string theory, which corresponds to the large  $N$  and strongly coupled limit of dual gauge theories. Therefore it is very intriguing to see how this formula is modified in the presence of corrections. In string theory, there are two quantum corrections: one is the quantum gravity corrections and the other is the stringy corrections as we mentioned. At present, we have little understanding on the former and thus here we will concentrate on the stringy corrections. These are described by higher derivative corrections to the Einstein gravity. Even for them the understanding is currently limited, the holographic entanglement entropy has been found only for the Lovelock gravities [104, 105] (see also later developments [81, 106]). Let us briefly review this in the simplest example: Gauss-Bonnet gravity. Its gravity action looks like

$$S_{GB} = -\frac{1}{16G_N} \int dx^{d+2} \sqrt{g} \left[ R - 2\Lambda + \lambda(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2) \right], \quad (16)$$

where  $\Lambda < 0$  is the negative cosmological constant of AdS space and  $\lambda$  is the Gauss-Bonnet parameter. The holographic entanglement entropy is argued to be [104, 105]

$$S_A = \text{Min}_{\Sigma_A} \left[ \frac{1}{4G_N} \int_{\Sigma_A} dx^d \sqrt{h} (1 + 2\lambda R_{int}) \right], \quad (17)$$

where  $R_{int}$  is the intrinsic curvature of  $\Sigma_A$ . This formula passes several non-trivial tests. See also [49, 107] for other aspects of higher derivative corrections to the holographic entanglement entropy.

## 3. Black Hole Formations and Quantum Quenches

So far we discussed the AdS/CFT at zero temperature or equally at a ground state. If we heat up the system, the CFT reaches to a thermal equilibrium state at a finite temperature. This finite temperature CFT is dual to a black hole in the AdS space [76]. The AdS/CFT in this case nicely fits with the well-know fact that the black hole follows thermodynamics. Moreover if we consider the process of the heating up the system,

where the temperature getting increasing, the gravity dual is described by a black hole formation in the AdS space. It is quite remarkable that the AdS/CFT allows us to analyze strongly coupled non-equilibrium systems. Below we would like to discuss the behavior of the holographic entanglement in these situations.

### 3.1. Holographic Entanglement Entropy at Finite Temperature

Consider a calculation of the holographic entanglement entropy at finite temperature  $T = \beta^{-1}$  in the simplest example of AdS/CFT i.e. the AdS<sub>3</sub>/CFT<sub>2</sub> duality. We assume that the spatial length of the total system  $L$  is infinite i.e.  $\beta/L \ll 1$ . In such a high temperature region, the gravity dual of the conformal field theory is described by the Euclidean BTZ black hole [108]. Its metric looks like

$$ds^2 = (r^2 - r_+^2)d\tau^2 + \frac{R^2}{r^2 - r_+^2}dr^2 + r^2d\varphi^2. \quad (18)$$

Note that if we set  $z = 1/r$  and perform trivial coordinate rescalings, we can confirm that this metric approaches to the pure AdS<sub>3</sub> (12) in the  $r \rightarrow \infty$  limit.

The Euclidean time is compactified as  $\tau \sim \tau + \frac{2\pi R}{r_+}$  to obtain a smooth geometry. We also impose the periodicity  $\varphi \sim \varphi + 2\pi$ . By taking the boundary limit  $r \rightarrow \infty$ , we find the relation between the boundary CFT and the geometry (18)

$$\frac{\beta}{L} = \frac{R}{r_+} \ll 1. \quad (19)$$

The subsystem for which we consider the entanglement entropy is given by  $0 \leq \varphi \leq 2\pi l/L$  at the boundary. Then by extending our formula (8) to asymptotically AdS spaces, the entropy can be computed from the length of the space-like geodesic starting from  $\varphi = 0$  and ending at  $\varphi = 2\pi l/L$  at the boundary  $r = r_0 \rightarrow \infty$  at a fixed time. This geodesic distance can be found analytically as

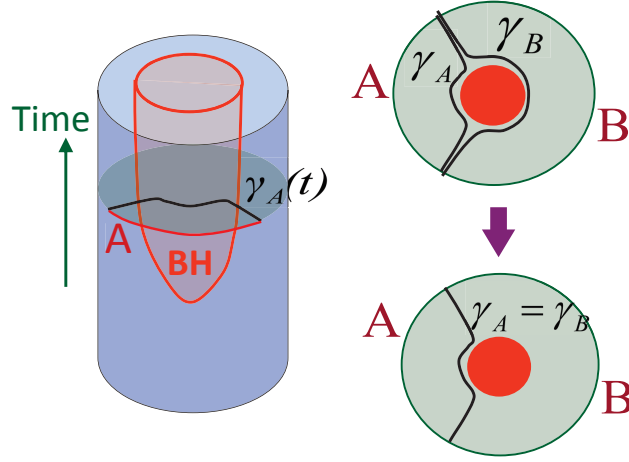
$$\cosh\left(\frac{\text{Length}(\gamma_A)}{R}\right) = 1 + \frac{2r_0^2}{r_+^2} \sinh^2\left(\frac{\pi l}{\beta}\right). \quad (20)$$

The relation between the cut off  $a$  in CFT and the one  $r_0$  of AdS is given by  $\frac{r_0}{r_+} = \frac{\beta}{a}$ . Then it is easy to see that our area law (8) precisely reproduces the known CFT result [6, 7] given by the following formula [38, 21, 22]

$$S_A = \frac{c}{3} \log\left(\frac{\beta}{\pi a} \sinh\left(\frac{\pi l}{\beta}\right)\right). \quad (21)$$

It is also useful to understand these calculations geometrically. The geodesic line in the BTZ black hole takes the form shown in the right upper picture in Fig. 4. When the size of  $A$  is small, it is almost the same as the one in the ordinary AdS<sub>3</sub>. As the size becomes large, the turning point approaches the horizon and eventually, the geodesic line covers a part of the horizon. This is the reason why we find a thermal extensive behavior of the entropy when  $l/\beta \gg 1$  in (21). The thermal entropy in a conformal field theory is dual to the black hole entropy in its gravity description via the AdS/CFT correspondence. In the presence of a horizon, it is clear that  $S_A$  is not

equal to  $S_B$  (remember  $B$  is the complement of  $A$ ) since the corresponding geodesic lines wrap different parts of the horizon (see the right upper picture in Fig. 4). This is a typical property of the entanglement entropy for a mixed state and thus the topological obstruction due to the black hole horizon directly corresponds to the basic property of mixed states.



**Figure 4.** The left figure schematically describes the black hole creation and the extremal surface  $\gamma_A$ . The orange curve represents the time-evolving black hole. The right figures describe how the extremal surface  $\gamma_A$  and  $\gamma_B$  should be chosen. In the black hole creation spacetime, the right lower picture describes the correct choice i.e.  $\gamma_A$  and  $\gamma_B$  coincides. On the other hand, for eternal blackholes (i.e time-independent black holes), we need to distinguish  $\gamma_A$  and  $\gamma_B$  as in the right upper figure.

### 3.2. Holographic Entanglement Entropy and Black Hole Formations

A more interesting backgrounds in AdS/CFT is time-dependent solutions where a black hole is formed [109, 110, 111, 112, 113]. A simple class of such examples of time-dependent backgrounds in QFTs is called quantum quenches [114, 115, 116]. A quantum quench is triggered by a sudden shift of parameters such as the mass in a quantum field theory. This means that the injection of energy is taken place instantly, shifting a ground state into an excited state at a given time. If the theory at later time is massless, we can regard the system as an excited state in a CFT. One of the important quantities which characterize such a time evolution is the entanglement entropy. As shown in [114], the entanglement entropy under a quantum quench in two dimensional CFTs always increases linearly as a function of time and eventually reaches a constant value after the thermalization time  $\Delta t$  as sketched in Fig.5. The increased amount of the entanglement entropy at late time  $t > \Delta t$  is the same as the thermal entropy at the final thermal equilibrium. The thermalization time  $\Delta t$  is found to be a half of the length  $l$  of the

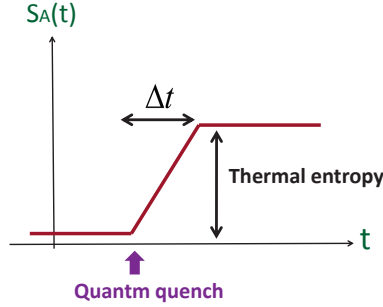
subsystem  $A$  [114], which is explained by assuming that the information propagates at a speed of light.

Analysis of the time evolution of entanglement entropy has started in [8] by employing the covariant holographic entanglement entropy, where the entangling surface  $\gamma_A$  is given by an extremal surface in the AdS space. Recently, there have been remarkable developments on studies of time-dependent holographic entanglement entropy [117, 118, 119, 120, 121, 123, 122, 124, 125, 126, 127, 128, 129]. For example, it has been confirmed that the holographic analysis agrees with the two dimensional CFT result [117]. In higher dimensional CFTs, the holographic analysis reveals that the thermalization time  $\Delta t$  depends on the shape of the subsystem  $A$  [118, 120, 127]. When  $A$  is a round ball with the radius  $l$ , then we have  $\Delta t = \frac{l}{2}$ , while when  $A$  is an infinitely extended strip with the width  $l$  we find  $\Delta t > \frac{l}{2}$ . The consistency with the strong subadditivity has been confirmed for various time-dependent examples in [124, 129]. Interesting oscillating modes have been found in [122]. The time evolution of holographic entanglement entropy under local quenches has been studied in [128].

Moreover, the holographic entanglement entropy allows us to answer the basic puzzle on the entropy of time-dependent black holes [117, 119]. In a thermalization of CFT, an initially pure state gets excited and evolves until it reaches the thermal equilibrium. Its gravity dual is a black hole formation in the AdS space. At early time, the spacetime is the pure AdS, while at late time, it approaches a static AdS black hole. Thus one might be tempting to conclude that the entropy, which is initially vanishing, should increase under the time evolution. This clearly contradicts with its CFT dual, where a pure state should follow a unitary evolution, which does not change the microscopic entropy. The total entropy  $S_{tot} = -\text{Tr} \rho \log \rho$  is conveniently calculated from the difference of the entanglement entropy

$$S_{tot} = \lim_{|B| \rightarrow 0} (S_A - S_B). \quad (22)$$

Indeed, for a pure state, which always satisfies  $S_A = S_B$ , we find  $S_{tot} = 0$  by using this formula. Then the holographic analysis explained in the Fig.4 shows that the total entropy is actually vanishing at any time. In this case, the presence of horizon is not a topological obstruction as we can modify the surfaces  $\gamma_A$  and  $\gamma_B$  so that it topologically equivalent. In this way, we can conclude that during a black hole formulation, the microscopic entropy does not increase [117, 119]. However, the coarse-grained entropy is increasing as its apparent horizon expands. Actually, we can regard the entanglement entropy  $S_A$  as a coarse-grained entropy because we trace out some part of the space and  $S_A$  indeed increases under the time evolution by the amount of the thermal entropy in the final equilibrium. Notice that there is no obvious unique definition for the entropy of a time-dependent black holes, while the definition of holographic entanglement entropy is unique even for time-dependent backgrounds.



**Figure 5.** The time evolution of the entanglement entropy in a two dimensional CFT.

#### 4. Holographic Dual of BCFT

As a final topic we would like to consider the holographic dual of CFT defined on a manifold  $M$  with a boundary  $\partial M$ , which is so called boundary conformal field theory (BCFT). This is argued to be given by generalizing the AdS/CFT correspondence in the following way [130, 131] (called as AdS/BCFT). Based on the idea of holography and AdS/CFT, we extend a  $d + 1$  dimensional manifold  $M$  to a  $d + 2$  dimensional asymptotically AdS space  $N$  so that  $\partial N = M \cup Q$ , where  $Q$  is a  $d + 1$  dimensional manifold which satisfies  $\partial Q = \partial M$ . See Fig.6 for this setup.

Usually, we impose the Dirichlet boundary condition on the metric at the boundary of AdS and following this we assume the Dirichlet boundary condition on  $M$ . On the other hand, we require a Neumann boundary condition on the metric at  $Q$ , whose details will be explained later. This change of boundary condition is the most important part of the holographic construction of BCFT.

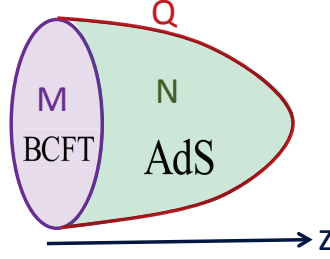
In specific setups, such a holography construction of BCFT has already been mentioned in the earlier papers [132, 133]. Different constructions of holographic dual of field theories with boundaries can be found in [134, 135, 136]. Moreover, our setup can be regarded as a modification of the well-known Randall-Sundrum setup [137] such that the additional boundary  $Q$  intersects with the original asymptotically AdS boundary.

##### 4.1. Construction

To make the variational problem sensible, we add the Gibbons-Hawking boundary term [138] to the Einstein-Hilbert action (we omit the boundary term for  $M$ ):

$$I = \frac{1}{16\pi G_N} \int_N \sqrt{-g}(R - 2\Lambda) + \frac{1}{8\pi G_N} \int_Q \sqrt{-h}K. \quad (23)$$

The metric of  $N$  and  $Q$  are denoted by  $g$  and  $h$ , respectively.  $K = h^{ab}K_{ab}$  is the trace of extrinsic curvature  $K_{ab}$  defined by  $K_{ab} = \nabla_a n_b$ , where  $n$  is the unit vector normal to  $Q$  with a projection of indices onto  $Q$  from  $N$ .



**Figure 6.** A sketch of general setups of holographic dual of BCFT

Consider the variation of metric in the above action. After a partial integration, we find

$$\delta I = \frac{1}{16\pi G_N} \int_Q \sqrt{-h} (K_{ab} \delta h^{ab} - K h_{ab} \delta h^{ab}). \quad (24)$$

Notice that the terms which involve the derivative of  $\delta h_{ab}$  cancels out thanks to the boundary term. We can add to (23) the action  $I_Q$  of some matter fields localized on  $Q$ . We impose the Neumann boundary condition instead of the Dirichlet one by setting the coefficients of  $\delta h^{ab}$  to zero and finally we obtain the boundary condition

$$K_{ab} - h_{ab} K = 8\pi G_N T_{ab}^Q, \quad (25)$$

where we defined

$$T^{Qab} = \frac{2}{\sqrt{-h}} \frac{\delta I_Q}{\delta h_{ab}}. \quad (26)$$

As a simple example we would like to assume that the boundary matter lagrangian is just a constant. This leads us to consider the following action

$$I = \frac{1}{16\pi G_N} \int_N \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G_N} \int_Q \sqrt{-h} (K - T). \quad (27)$$

The constant  $T$  is interpreted as the tension of the boundary surface  $Q$ . In AdS/CFT, a  $d + 2$  dimensional AdS space ( $\text{AdS}_{d+2}$ ) is dual to a  $d + 1$  dimensional CFT. The geometrical  $SO(2, d + 1)$  symmetry of AdS is equivalent to the conformal symmetry of the CFT. When we put a  $d$  dimensional boundary to a  $d + 1$  dimensional CFT such that the presence of the boundary breaks  $SO(2, d + 1)$  into  $SO(2, d)$ , this is called a boundary conformal field theory (BCFT) [139]. Note that though the holographic duals of defect or interface CFTs [132, 133, 140] look very similar with respect to the symmetries, their gravity duals are different from ours because they do not have extra boundaries like  $Q$ .

To realize this structure of symmetries, we take the following ansatz of the metric (see also [132, 133, 141]):

$$ds^2 = d\rho^2 + \cosh^2 \frac{\rho}{R} \cdot ds_{\text{AdS}_{d+1}}^2. \quad (28)$$



If we assume that  $\rho$  takes all values from  $-\infty$  to  $\infty$ , then (28) is equivalent to the  $\text{AdS}_{d+2}$ . To see this, let us assume the Poincare metric of  $\text{AdS}_{d+1}$  by setting

$$ds^2_{\text{AdS}_{d+1}} = R^2 \frac{-dt^2 + dy^2 + d\vec{w}^2}{y^2}, \quad (29)$$

where  $\vec{w} \in R^{d-1}$ . Remember that the cosmological constant  $\Lambda$  is related to the AdS radius  $R$  by  $\Lambda = -\frac{(d+1)d}{2R^2}$ .

By defining new coordinates  $z$  and  $x$  as

$$z = y / \cosh \frac{\rho}{R}, \quad x = y \tanh \frac{\rho}{R}, \quad (30)$$

we recover the familiar form of the Poincare metric of  $\text{AdS}_{d+2}$ :  $ds^2 = R^2(dz^2 - dt^2 + dx^2 + d\vec{w}^2)/z^2$ .

To realize a gravity dual of BCFT, we will put the boundary  $Q$  at  $\rho = \rho_*$  and this means that we restrict the spacetime to the region  $-\infty < \rho < \rho_*$ . The extrinsic curvature on  $Q$  reads

$$K_{ab} = \frac{1}{R} \tanh \left( \frac{\rho}{R} \right) h_{ab}. \quad (31)$$

The boundary condition (25) leads to

$$K_{ab} = (K - T)h_{ab}. \quad (32)$$

Thus  $\rho_*$  is determined by the tension  $T$  as follows

$$T = \frac{d}{R} \tanh \frac{\rho_*}{R}. \quad (33)$$

#### 4.2. Boundary Entropy

Let us concentrate on the  $d = 1$  case to describe the two dimensional BCFT. This setup is special in that it has been well-studied (see [142] and references therein) and that the BCFT has an interesting quantity called the boundary entropy (or  $g$ -function) [143]. We define the quantity called  $g$  by the partition function on a disk denoted by  $g_\alpha$ , where  $\alpha$  parameterizes the choice of boundary conditions. The boundary entropy  $S_{bdy}^{(\alpha)}$  is defined by

$$S_{bdy}^{(\alpha)} = \log g_\alpha. \quad (34)$$

The boundary entropy measures the boundary degrees of freedom and can be regarded as a boundary analogue of the central charge  $c$ .

Consider a holographic dual of a CFT on a round disk defined by  $\tau^2 + x^2 \leq r_D^2$  in the Euclidean  $\text{AdS}_3$  spacetime

$$ds^2 = R^2 \frac{dz^2 + d\tau^2 + dx^2}{z^2}, \quad (35)$$

where  $\tau$  is the Euclidean time. In the Euclidean formulation, the action (27) is now replaced by

$$I_E = -\frac{1}{16\pi G_N} \int_N \sqrt{g}(R - 2\Lambda) - \frac{1}{8\pi G_N} \int_Q \sqrt{h}(K - T). \quad (36)$$

Note that  $\rho_*$  is related to the tension  $T$  of the boundary via (33). When the BCFT is defined on the half space  $x < 0$ , its gravity dual has been found in previous subsection. Therefore we can find the gravity dual of the BCFT on the round disk by applying the conformal map (see e.g. [144]). The final answer is the following domain in  $\text{AdS}_3$

$$\tau^2 + x^2 + (z - \sinh(\rho_*/R)r_D)^2 - r_D^2 \cosh^2(\rho_*/R) \leq 0. \quad (37)$$

In this way we found that the holographic dual of BCFT on a round disk is given by a part of the two dimensional round sphere. A larger value of tension corresponds to the larger radius.

Now we would like to calculate the disk partition function in order to obtain the boundary entropy. By evaluating (36) in the domain (37), we obtain

$$I_E = \frac{R}{4G_N} \left( \frac{r_D^2}{2a^2} + \frac{r_D \sinh(\rho_*/R)}{a} + \log(a/r_D) - \frac{1}{2} - \frac{\rho_*}{R} \right), \quad (38)$$

where we introduced the UV cutoff  $z > a$  as before. By adding the counter term on the AdS boundary [145], we can subtract the divergent terms in (38). The difference of the partition function between  $\rho = 0$  and  $\rho = \rho_*$  is given by  $I_E(\rho_*) - I_E(0) = -\frac{\rho_*}{4G_N}$ . Since the partition function is given by  $Z = e^{-I_E}$ , we obtain the boundary entropy

$$S_{bdy} = \frac{\rho_*}{4G_N}, \quad (39)$$

where we assumed  $S_{bdy} = 0$  for  $T = 0$  because the boundary contributions vanish in this case.

Another way to extract the boundary entropy is to calculate the entanglement entropy. In a two dimensional CFT on a half line,  $S_A$  behaves as follows [38, 21, 22]

$$S_A = \frac{c}{6} \log \frac{l}{a} + \log g, \quad (40)$$

where  $c$  is the central charge and  $a$  is the UV cut off (or lattice spacing);  $A$  is chosen to be an interval with length  $l$  such that it ends at the boundary. The  $\log g$  in (40) coincides with the boundary entropy (34).

In AdS/CFT, the holographic entanglement entropy can be calculated by the formula (2). Consider the gravity dual of a two dimensional BCFT on a half line  $x < 0$  in the coordinate (35). By taking the time slice  $\tau = 0$ , we define the subsystem  $A$  by the interval  $-l \leq x \leq 0$ . In this case, the minimal surface (or geodesic line)  $\gamma_A$  is given by  $x^2 + z^2 = L^2$ . If we go back to the coordinate system (28) and (29), then  $\gamma_A$  is simply given by  $\tau = 0, y = l$  and  $-\infty < \rho \leq \rho_*$ . This leads to

$$S_A = \frac{1}{4G_N} \int_{-\infty}^{\rho_*} d\rho. \quad (41)$$

By subtracting the bulk contribution which is divergent as in (40), we reproduce the previous result (39). See also [136] for the recent calculation of boundary entropy in supergravity. A similar calculations of boundary entropy for interface CFTs can be found in [146].

### 4.3. Holographic g-theorem

In two dimension, the central charge  $c$  is the most important quantity which characterizes the degrees of freedom of CFT. Moreover, there is a well-known fact, so called c-theorem [147], that the central charge monotonically decreases under the RG flow. In the case of BCFT, an analogous quantity is actually known to be the g-function or equally boundary entropy [143]. At fixed points of boundary RG flows, it is reduced to that of BCFT introduced in (34). It has been conjectured that the g-function monotonically decreases under the boundary RG flow in [143] and this has been proven in [148] later. Therefore the holographic proof of g-theorem described below will offer us an important evidence of our proposed holography. Refer to [149] for a holographic c-theorem and to [150] for a holographic g-theorem in the defect CFT under a probe approximation.

Because we want to keep the bulk conformal invariance and we know that all solutions to the vacuum Einstein equation with  $\Lambda < 0$  are locally  $\text{AdS}_3$ , we expect that the bulk spacetime remains to be  $\text{AdS}_3$ . We describe the boundary  $Q$  by the curve  $x = x(z)$  in the metric (35). We assume generic matter fields on  $Q$  and this leads to the energy stress tensor  $T_{ab}^Q$  term in the boundary condition (25). It is easy to check the energy conservation  $\nabla^a T_{ab}^Q = 0$  in our setup because  $\nabla^a (K_{ab} - K h_{ab}) = R_{nb}$ , where  $n$  is the Gaussian normal coordinate which is normal to  $Q$ . In order to require that the matter fields on the boundary are physically sensible, we impose the null energy condition (or weaker energy condition) as in the holographic c-theorem [149]. It is given by the following inequality for any null vector  $N^a$

$$T_{ab}^Q N^a N^b \geq 0. \quad (42)$$

In our case, we can choose

$$(N^t, N^z, N^x) = \left( \pm 1, \frac{1}{\sqrt{1 + (x'(z))^2}}, \frac{x'(z)}{\sqrt{1 + (x'(z))^2}} \right). \quad (43)$$

Then the condition (42) is equivalent to

$$x''(z) \leq 0. \quad (44)$$

Since at a fixed point the boundary entropy is given by  $S_{bdy} = \frac{\rho_*}{4G_N}$  and we have the relation  $\frac{x}{z} = \sinh(\rho_*/R)$  on the boundary  $Q$ , we would like to propose the following g-function

$$\log g(z) = \frac{R}{4G_N} \cdot \text{arcsinh} \left( \frac{x(z)}{z} \right). \quad (45)$$

By taking derivative, we get

$$\frac{\partial \log g(z)}{\partial z} = \frac{x'(z)z - x(z)}{\sqrt{z^2 + x(z)^2}}. \quad (46)$$

Indeed we can see that  $x'z - x$  is non-positive because this is vanishing at  $z = 0$  and (44) leads to  $(x'z - x)' = x''z \leq 0$ . Thus we can show that  $g(z)$  is a monotonically

decreasing function of  $z$ , which is dual to the length scale of the dual BCFT. In this way, we manage to derive the g-theorem in our setup. We can generalize this argument into higher dimensions [131], which leads to a proposal of a higher dimensional analogue of the g-theorem. Refer to [151, 152, 153, 154, 155] for other aspects of AdS/BCFT.

## 5. Conclusions

In this review article, we presented a quick survey on the recent progresses on holographic entanglement entropy (HEE). We can think of several applications of HEE to various subjects. One of them will be quantum mechanical understandings of black holes, which was historically the original motivation of considering the entanglement entropy in quantum field theories. For example, we explained that the HEE can give a useful order parameter for black hole formation processes which are dual to the thermalization of strongly coupled systems. This is expected to give a nice relation between the black hole physics and non-equilibrium physics. Refer also to [156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169] for other progresses on the HEE for black holes which we could not discuss in the main context of this article. For applications of holographic entanglement entropy to brane-world setups refer to [170, 171, 172] (see also [173]), which was pioneered by [19]. Also, there have been studies of holography in non-trivial spacetimes such as flat space [174, 175] and AdS wormholes [176, 177].

The applications of HEE to condensed matter physics is also very intriguing. For example, the HEE is employed to search gravity duals with Fermi surfaces [178, 179, 180, 181, 182, 183]. An interesting behavior analogous to the entanglement entropy has been observed in a problem of image compression [184]. It has been pointed out that the holographic entanglement entropy supports the idea of emergent gravity in [185] (see also [186]). One way to make this idea concrete seems to employ the conjectured connection [187] between the AdS/CFT and the multi scale entanglement renormalization (MERA) [188] (see also [189]).

Then it is natural to ask how the quantum information in CFT is encoded in the AdS spacetime. It is well expected that the entanglement entropy will play an important role again here. The methods to extract the bulk metric from then holographic entanglement entropy has been discussed in [190, 191, 127]. Moreover, quite recently, there have been interesting discussions on reconstructions of the bulk geometry from the information on a certain region at the boundary [192, 193, 194]. Indeed, the idea of holographic entanglement entropy has turned out to be closely related [193, 194] and more detailed analysis certainly deserves a future study.

In this way, the entanglement entropy connects directly between gravity backgrounds and quantum states in quantum many-body systems. Though the metric in gravity may not be a good quantity to look at in the presence of significant quantum corrections, the (holographic) entanglement entropy should be well-defined even at the quantum level. Thus the HEE should be useful for the understanding of both quantum

gravity and condensed matter systems.

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